

NAG Toolbox for MATLAB

f07pr

1 Purpose

f07pr computes the Bunch–Kaufman factorization of a complex Hermitian indefinite matrix, using packed storage.

2 Syntax

```
[ap, ipiv, info] = f07pr(uplo, n, ap)
```

3 Description

f07pr factorizes a complex Hermitian matrix A , using the Bunch–Kaufman diagonal pivoting method and packed storage. A is factorized as either $A = PUDU^H P^T$ if **uplo** = 'U' or $A = PLDL^H P^T$ if **uplo** = 'L', where P is a permutation matrix, U (or L) is a unit upper (or lower) triangular matrix and D is an Hermitian block diagonal matrix with 1 by 1 and 2 by 2 diagonal blocks; U (or L) has 2 by 2 unit diagonal blocks corresponding to the 2 by 2 blocks of D . Row and column interchanges are performed to ensure numerical stability while keeping the matrix Hermitian.

This method is suitable for Hermitian matrices which are not known to be positive-definite. If A is in fact positive-definite, no interchanges are performed and no 2 by 2 blocks occur in D .

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – string

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo = 'U'

The upper triangular part of A is stored and A is factorized as $PUDU^H P^T$, where U is upper triangular.

uplo = 'L'

The lower triangular part of A is stored and A is factorized as $PLDL^H P^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **n** – int32 scalar

n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **ap**(*) – complex array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

The n by n Hermitian matrix A , packed by columns.

More precisely,

if **uplo** = 'U', the upper triangle of A must be stored with element A_{ij} in **ap**($i + j(j - 1)/2$) for $i \leq j$;
 if **uplo** = 'L', the lower triangle of A must be stored with element A_{ij} in **ap**($i + (2n - j)(j - 1)/2$) for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**(*) – complex array

Note: the dimension of the array **ap** must be at least $\max(1, n \times (n + 1)/2)$.

A contains details of the block diagonal matrix D and the multipliers used to obtain the factor U or L as specified by **uplo**.

2: **ipiv**(*) – int32 array

Note: the dimension of the array **ipiv** must be at least $\max(1, n)$.

Details of the interchanges and the block structure of D . More precisely,

if **ipiv**(i) = $k > 0$, d_{ii} is a 1 by 1 pivot block and the i th row and column of A were interchanged with the k th row and column;

if **uplo** = 'U' and **ipiv**($i - 1$) = **ipiv**(i) = $-l < 0$, $\begin{pmatrix} d_{i-1,i-1} & \bar{d}_{i,i-1} \\ \bar{d}_{i,i-1} & d_{ii} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i - 1)$ th row and column of A were interchanged with the l th row and column;

if **uplo** = 'L' and **ipiv**(i) = **ipiv**($i + 1$) = $-m < 0$, $\begin{pmatrix} d_{ii} & d_{i+1,i} \\ d_{i+1,i} & d_{i+1,i+1} \end{pmatrix}$ is a 2 by 2 pivot block and the $(i + 1)$ th row and column of A were interchanged with the m th row and column.

3: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **ipiv**, 5: **info**.

info > 0

If **info** = i , $d(i, i)$ is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by zero will occur if it is used to solve a system of equations.

7 Accuracy

If **uplo** = 'U', the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon P|U||D||U^H|P^T,$$

$c(n)$ is a modest linear function of n , and ϵ is the *machine precision*.

If **uplo** = 'L', a similar statement holds for the computed factors L and D .

8 Further Comments

The elements of D overwrite the corresponding elements of A ; if D has 2 by 2 blocks, only the upper or lower triangle is stored, as specified by **uplo**.

The unit diagonal elements of U or L and the 2 by 2 unit diagonal blocks are not stored. The remaining elements of U and L are stored in the corresponding columns of the array **ap**, but additional row interchanges must be applied to recover U or L explicitly (this is seldom necessary). If **ipiv**(i) = i , for $i = 1, 2, \dots, n$ (as is the case when A is positive-definite), then U or L are stored explicitly in packed form (except for their unit diagonal elements which are equal to 1).

The total number of real floating-point operations is approximately $\frac{4}{3}n^3$.

A call to f07pr may be followed by calls to the functions:

f07ps to solve $AX = B$;

f07pu to estimate the condition number of A ;

f07pw to compute the inverse of A .

The real analogue of this function is f07pd.

9 Example

```

uplo = 'L';
n = int32(4);
ap = [complex(-1.36, +0);
      complex(1.58, -0.9);
      complex(2.21, +0.21);
      complex(3.91, -1.5);
      complex(-8.869999999999999, +0);
      complex(-1.84, +0.03);
      complex(-1.78, -1.18);
      complex(-4.63, +0);
      complex(0.11, -0.11);
      complex(-1.84, +0)];
[apOut, ipiv, info] = f07pr(uplo, n, ap)

apOut =
-1.3600
 3.9100 - 1.5000i
 0.3100 + 0.0433i
-0.1518 + 0.3743i
-1.8400
 0.5637 + 0.2850i
 0.3397 + 0.0303i
-5.4176
 0.2997 + 0.1578i
-7.1028
ipiv =
    -4
    -4
     3
     4
info =
```

0